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Pull-in voltage of microswitch rough plates in the presence of electromagnetic and acoustic Casimir forces

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In this work, we investigate the combined influence of electromagnetic and acoustic Casimir forces on the pull-in voltage of microswitches with self-affine rough plates. It is shown that for plate separations within the micron range the acoustic term arising from pressure fluctuations can influence significantly the pull-in potential in a manner that depends on the particular roughness characteristics. Indeed, the acoustic term contribution can be comparable to that of surface roughness. Moreover, the temperature influence from the acoustic term appears to play a significant role besides that arising from the temperature dependence of the electromagnetic force due to quantum vacuum fluctuations. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2711409]

I. INTRODUCTION

Diverse micro-/nanoelectromechanical systems (MEMSs/NEMSs) make use of microswitches as an essential operation component and offer access to an unprecedented parameter space for sensing and fundamental measurements.[1–14] A typical microswitch is constructed from two conducting electrodes. One electrode is usually able to move suspended by a mechanical spring. By applying a voltage difference between the two electrodes, the mobile electrode moves towards the ground electrode due to the electrostatic force. However, at a certain voltage the mobile electrode becomes unstable and collapses or pulls in on the fixed ground electrode.[3,4] The pull-in properties can also be strongly influenced by forces between neutral bodies in close approach. Indeed, when the proximity between the plates of switches becomes of the order of nanometers up to a few microns, a regime is entered where forces that are quantum mechanical in nature, namely, van der Waals (vdW) and Casimir forces, become operative.[15–18] These forces may be responsible for stiction by causing mechanical elements in close proximity to adhere together, and can also alter the actuation dynamics of switches.[9]

Especially the Casimir force has been considered to be an exotic quantum phenomenon that results from the perturbation of zero point eletromagnetic (EM) vacuum fluctuations by the presence of conducting plates.[15–18] Because of its relatively short range (for separations >50 nm), it is now starting to attract technological importance for the design and operation of MEMS/NEMS.[9–13] Recent studies for switches with rough plates have shown that random self-affine roughness, which often occurs during nonequilibrium film growth, strongly influence pull-in parameters of microswitches in presence of electrostatic, Casimir, and capillary forces.[18,19]

Besides EM vacuum fluctuations which induce an attractive Casimir force,[15] Larraza[20] transferred this idea into acoustics and measured the force between two parallel plates in an external sound field with a bandwidth from 5 to 20 kHz.[20] In the space between the two plates, lower-frequency modes were suppressed leading to an attractive force. The force is also repulsive when the distance between the plates was comparable to the half-wavelength associated with the lower edge of the frequency band. Furthermore, for a gas at rest there are thermodynamical pressure fluctuations and Brownian motion. As a result the plates experience an attractive force per unit area $f_{\text{acou}}=\pi K_B T/18d^3$ with d as the plate separation and T as the gas temperature.[21]

If someone compares the acoustic pressure f_{acou} with the EM Casimir pressure for flat plate surfaces $f_{\text{Cas}}=\pi hc/480d^4$ (with c the velocity of light), it is obtained that $f_{\text{Cas}}\approx f_{\text{acou}}$ for $d=1800$ nm at $T=300$ K.[21] This is an accessible size during fabrication of microswitches, and comparable to the range where temperature corrections are significant on the EM Casimir force. In this case, a typical thermal wavelength is $\lambda_T=\hbar c/2K_B T$ which at $T=300$ K yields $\lambda_T=6.55\ \mu$m. Therefore, since thermal fluctuations for $T\geqslant 300$ K are important at micron plate separations and produce their own radiation pressure on the EM Casmir force, the influence of corresponding acoustic force should also be thoroughly considered during the calculation of pull-in characteristics of electromechanical switches.

II. THEORY FOR SWITCHES WITH PARALLEL ROUGH PLATES

We consider a parallel plate configuration with the electrostatic and Casimir force pulling the plates together against an opposing elastic restoring force. The initial plate distance is d, the average flat plate area is A_{flat}, the plate spring constant is k and its mass is m, and the voltage across the plates is V. ε_0 is the dielectric constant of the medium between the plates. We also assume single valued roughness fluctuations $h(R)$ of the in-plane position $R=(x,y)$. The restoring force is given by[13,18]

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$$F_k = -k(d-r). \qquad (1)$$

The electrostatic force without accounting for fringing fields for a plate separation r ($\leq d$) and for a Gaussian height distribution is given by[13,18,22]

$$F_{\mathrm{el}} = \frac{\varepsilon_0 A_r}{2}\frac{V^2}{r^2}\int_0^{+\infty} e^{-y}\sqrt{1+\rho_{\mathrm{rms}}{}^2 y}\,dy. \qquad (2)$$

The integral in Eq. (2) gives area ratio $R_r = A_r/A_{\mathrm{flat}}$ where A_r is the rough surface area, and $\rho_{\mathrm{rms}} = \sqrt{\langle|\nabla h|^2\rangle}$ is the average local surface slope.[23] Furthermore, the Casimir force for rough plates (both with the same roughness profiles) is given by[18,24,25]

$$F_{\mathrm{Cas}}(T,r) \cong \frac{\pi hc}{480 r^4} A_{\mathrm{flat}}\left(1+\frac{2C_r}{r}\right)F^T(T,r). \qquad (3)$$

The roughness factor C_r and the temperature correction $F^T(T,r)$ are given, respectively, by[18]

$$C_r = \begin{cases} 0.4492\displaystyle\int_{Q_r}^{Q_c} q\langle|h(q)|^2\rangle\frac{d^2q}{(2\pi)^2} & \text{if } r<\lambda_p \\ \frac{1}{3}\displaystyle\int_{Q_r}^{Q_{\lambda_p}} q\langle|h(q)|^2\rangle\frac{d^2q}{(2\pi)^2} + \frac{7}{15\pi}\frac{\lambda_p}{r}\int_{Q_{\lambda_p}}^{Q_c} q\langle|h(q)|^2\rangle\frac{d^2q}{(2\pi)^2} & \text{if } r>\lambda_p, \end{cases} \qquad (4)$$

$$F^T(T,r) = \begin{cases} 1+\dfrac{720}{\pi^2}\left[\left(\dfrac{K_BTr}{\hbar c}\right)^3\dfrac{\zeta(3)}{2\pi} - \dfrac{45}{\pi^2}\left(\dfrac{K_BTr}{\hbar c}\right)^4\right] & \text{if } K_BTr/\hbar c<1/2 \\ \left(\dfrac{K_BTr}{\hbar c}\right)\dfrac{\zeta(3)}{8\pi} - \dfrac{\pi^2}{720} & \text{if } K_BTr/\hbar c>1/2, \end{cases} \qquad (5)$$

where $\zeta(3)\approx 1.202$, $\langle|h(q)|^2\rangle$ is the roughness spectrum, $Q_{\lambda_P} = 2\pi/\lambda_P$, and $Q_r = 2\pi/r$, and λ_P is the finite plasmon wavelength (e.g., $\lambda_P \approx 100$ nm for Al). The combined conductivity-roughness and temperature corrections can be treated independently and multiplied for theory estimations above the 1% accuracy.[25] This is because the conductivity-roughness correction varies strongly at separations $\sim\lambda_P$ ($\ll 1\ \mu$m), while the temperature correction varies at much larger separations $\sim\lambda_T$ ($\gg 1\ \mu$m).

Finally, we consider for the attractive acoustic force to be given approximately by the following the expression:[20]

$$F_{\mathrm{acou}} \cong \frac{\pi K_B T}{18 d^3}\int_0^{+\infty} e^{-y}\sqrt{1+\rho_{\mathrm{rms}}{}^2 y}\,dy, \qquad (6)$$

where morphology corrections are taking into account by the area roughness factor R_r. A more rigorous treatment must consider the full scattering problem of sound waves.[26]

Furthermore, the plate motion is described by the second law of Newton: $m(d^2r/dt^2) = |F_k| - |F_{\mathrm{el}}| - |F_{\mathrm{Cas}}| - |F_{\mathrm{acou}}|$. Changing the variables so that $u = r/d$ $(0<u<1)$, $M = m/kT^2$, $\tau = t/T$ (T a characteristic time), $\alpha = \pi^2\hbar A_f/kd^5$, $\beta = \varepsilon_0 A_f V^2/kd^3$, and $C = A_{\mathrm{flat}}(\pi K_B T)/18kd^4$, we obtain the following simpler form:

$$M\frac{d^2u}{d\tau^2} = f(u) = 1-u-\frac{\beta R_r}{2u^2} - \frac{a}{240}W(u) - \frac{CR_r}{u^3}, \qquad (7)$$

with $W(u) = u^{-4}[1+(2C_r/du)]F^T(T,du)$. In order to obtain the pull-in potential we set in Eq. (7) $f(u)=0$ and $df(u)/du = 0$. The solution of these conditions yields

$$1-u-\frac{\beta R_r}{2u^2} - \frac{a}{240}W(u) - \frac{CR_r}{u^3} = 0,$$

$$\text{and } -1+\frac{\beta R_r}{u^3} - \frac{a}{240}\dot{W}(u) + \frac{3CR_r}{u^4} = 0. \qquad (8)$$

The solution of Eqs. (8) with respect to $\beta = \varepsilon_0 A_f V^2/kd^3$ yields the following pull-in potential:

$$V_{\mathrm{PI}} = V_0 u\sqrt{2}\sqrt{\frac{1}{R_r}\left[1-u+\frac{W}{\dot{W}}+\frac{CR_r}{u^3}\left(\frac{3}{u}\frac{W}{\dot{W}}-1\right)\right]\left(1+\frac{2}{u}\frac{W}{\dot{W}}\right)^{-1}}, \qquad (9)$$

with $V_0 = \sqrt{kd^3/\varepsilon_0 A_f}$ and $\dot{W} = dW/du$.

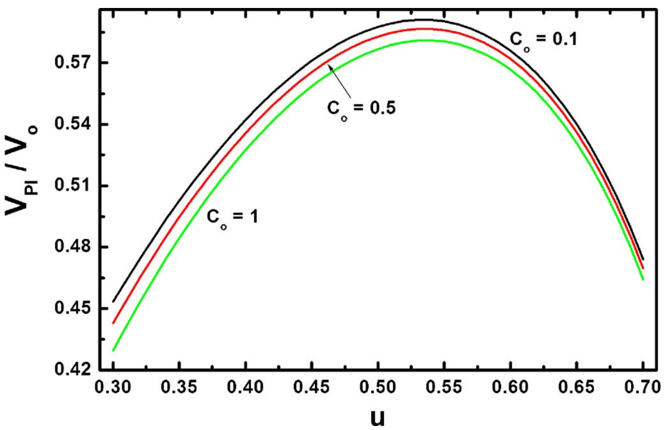

FIG. 1. (Color online) Pull-in voltage $V_{\rm PI}$ vs u for $a_0=0.3$ nm, $\lambda_P=100$ nm, $w=5$ nm, $\xi=200$ nm, $H=0.7$, plate separations $d=1000$ nm, acoustic constant $C=C_0 10^{-9}$ with C_0 *as* indicated, and $T=300$ K.

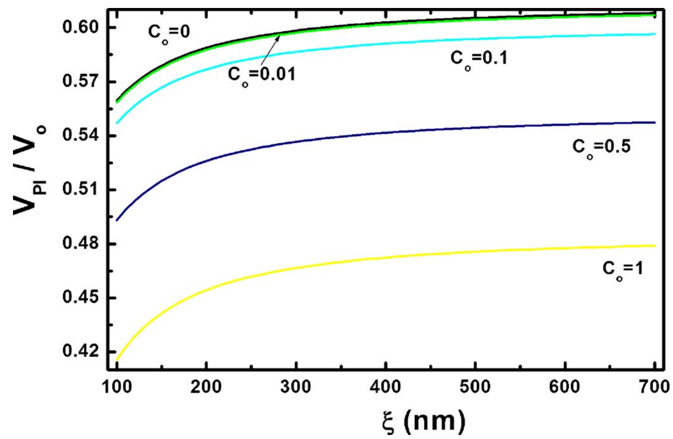

FIG. 2. (Color online) Pull-in voltage $V_{\rm PI}$ vs ξ for $a_0=0.3$ nm, $u=0.5$, $\lambda_P=100$ nm, $w=5$ nm, $H=0.7$, plate separations $d=1000$ nm, acoustic term constant $C=C_0 10^{-9}$ with C_0 *as* indicated, and $T=300$ K.

III. RESULTS AND DISCUSSION

Calculations of the pull-in potential from Eq. (9) requires knowledge of the roughness spectrum $\langle|h(q)|^2\rangle$. In fact, a diverse variety of surfaces that appear in thin films grown under nonequilibrium conditions possess the so-called self-affine roughness.[27] In this case the spectrum $\langle|h(q)|^2\rangle$ shows a power law scaling[27,28] $\langle|h(q)|^2\rangle \propto q^{-2-2H}$ if $q\xi \gg 1$ and $\langle|h(q)|^2\rangle \propto$ const if $q\xi \ll 1$. This is satisfied by the analytical model[28]

$$\langle|h(q)|^2\rangle = 2\pi \frac{w^2\xi^2}{(1+aq^2\xi^2)^{1+H}}, \qquad (10)$$

with $a=1/2H[1-(1+aQ_c{}^2\xi^2)^{-H}]$, $(0<H<1)$ $a=1/2\ln(1+aQ_c{}^2\xi^2)$ $(H=0)$. $Q_c=\pi/a_0$ with a_0 an atomic dimension lower roughness cut off. Small values of the roughness exponent H (~ 0) characterize jagged or irregular surfaces, while large values of H (~ 1) surfaces with smooth hills and valleys.[27,28] For other correlation models see Refs. 19 and 29. Upon substitution of Eq. (10) into the surface local slope expression $\rho_{\rm rms}=[\int_{0<q<Q_c} q^2\langle|h(q)|^2\rangle d^2q/(2\pi)^2]^{1/2}$, we obtain the simple analytic form $\rho_{\rm rms}=(w/\sqrt{2}\xi a)\sqrt{(1-H)^{-1}[(1+aQ_c{}^2\xi^2)^{1-H}-1]-2a}$.[23] The latter is useful in surface area calculations.[23]

Figure 1 shows calculations of the pull-in voltage $V_{\rm PI}$ versus the normalized separation u for various acoustic parameters C, and relatively large plate separation $d=1000$ nm $(\gg \lambda_P=100$ nm). The initial separation was chosen sufficiently large in order to be close to the separation $\tilde{d}=3hc/80K_BT$ $(f_{\rm Cas}\approx f_{\rm acou})$ where the acoustic and EM Casimir forces are of the same strength for the case of flat plate surfaces.[21] Notably for $T=300$ K we have $\tilde{d}=1800$ nm. By increasing the strength of the acoustic force, or increasing the constant C, the pull-in potential decreases for separations lower than the initial separation or $u<1$. From Eq. (9) it can be seen that the pull-in voltage is becoming zero $V_{\rm PI}=0$ for separations given by the complex equation $1-u+g(u)+[(3g(u)/u)-1](CR_r/u^3)=0$ where $g(u)=W(u)/\dot{W}(u)$. If we solve for the critical acoustic force constant C, above which the pull-in potential is zero, we obtain $\tilde{C}=u^3[1-u+g(u)]/\{[3g(u)/u]-1\}R_r$. The latter appears to depend predominantly inversely on the area ratio R_r, indicating its increment with decreasing surface roughness or equivalently decreasing R_r towards its asymptotic value for flat surfaces $(R_r \sim 1)$.

In order to gain further insight on the effect of the acoustic terms on the pull-in potential, Fig. 2 shows calculations of the pull-in potential versus correlation length ξ for various strengths of the acoustic parameter C. With increasing correlation length ξ, which implies smoothening at long wavelengths (for fixed roughness amplitude w), the pull-in potential increases. The increment is more pronounced with increasing acoustic strength C leading to lower pull-in potential in agreement also with Fig. 1. In addition, for increasing short wavelength roughness or decreasing roughness exponent H, the pull-in voltage decreases as it shown in Fig. 3. Similar also is the behavior of the pull-in voltage with increasing rms roughness amplitude w as it is shown in Fig. 4.

If we compare Figs. 2–4 it becomes evident that the effect of the roughness exponent H is very prominent even for variations within consecutive values close to the experimental uncertainty (typically from ±0.05 to ±0.1).[27] If we compare Figs. 2 and 3 we can infer that the influence of

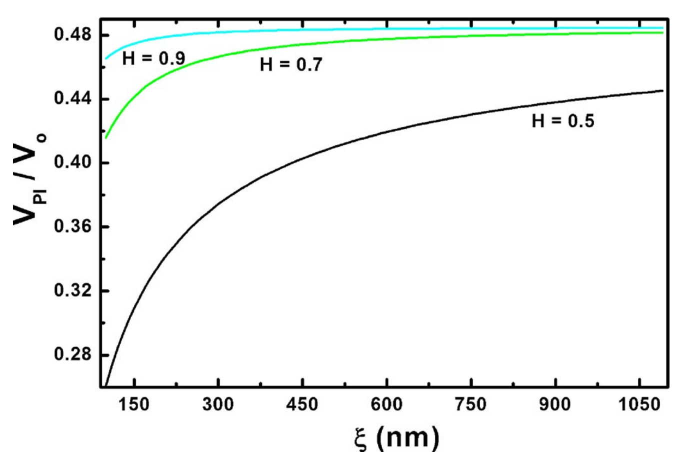

FIG. 3. (Color online) Pull-in voltage $V_{\rm PI}$ vs ξ for $a_0=0.3$ nm, $u=0.5$, $\lambda_P=100$ nm, $w=5$ nm, roughness exponent H as indicated, plate separations $d=1000$ nm, $C=C_0 10^{-9}$ $(C_0=1)$, and $T=300$ K.

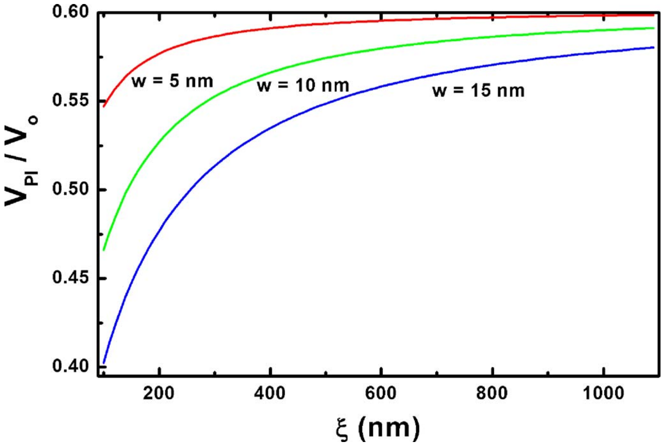

FIG. 4. (Color online) Pull-in voltage V_{PI} vs ξ for $a_0=0.3$ nm, $u=0.5$, $\lambda_P=100$ nm, $H=0.7$, plate separations $d=1000$ nm, acoustic term constant $C=C_0 10^{-9}$ ($C_0=1$), $T=300$ K, and rms roughness amplitude w as indicated.

decreasing roughness exponent H can lead to comparable decrement of the pull-in potential as that of increasing the strength C of the acoustic effect. Therefore, acoustic Casimir effects have to be taken carefully into account for switches with rough plates operating under environmental conditions, where pressure fluctuations lead to the acoustic forces.

Finally, since both EM and acoustic Casimir forces depend on temperature, we investigate in Fig. 5 the temperature influence of the pull-in potential. The temperature contributions arise for the EM Casimir force by the correction factor $F^T(T,r)$ in Eq. (5), while the acoustic contribution depends directly proportional to the system temperature from Eq. (6). The high temperature limit in Eq. (5) for the EM case, where $F^T(T,r) \sim T$, occurs for $T > T_{\mathrm{eff}}$ $(=\hbar c/2K_B ud)$. For plate separation, for example, $ud=500$ nm ($u=0.5$ and $d=1000$ nm) the transition temperature is $T_{\mathrm{eff}}=2290$ K, which is extremely high to be realized for any viable device. Therefore the acoustic term appears to have more dominant contribution when $T \ll T_{\mathrm{eff}}$. The latter becomes evident if one compares the curves in Fig. 5 with $300 \leqslant T \leqslant 900$ K. In any case, the temperature effect is more influential for large correlation lengths ξ or smoother surfaces as Fig. 5 indicates.

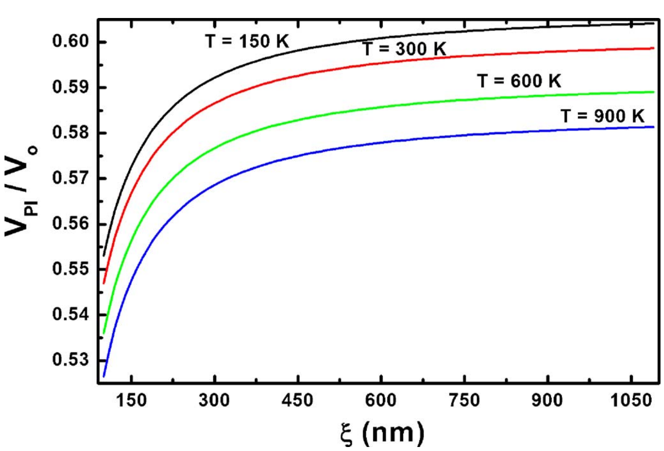

FIG. 5. (Color online) Pull-in voltage V_{PI} vs ξ for $a_0=0.3$ nm, $u=0.5$, $w=5$ nm, $\lambda_P=100$ nm, $H=0.7$, plate separations $d=1000$ nm, acoustic term constant $C=C_0 10^{-9}$ ($C_0=1$), and system temperatures T as indicated.

IV. CONCLUSIONS

In summary, we tried to gain further insight on the combined influence of random self-affine roughness and electromagnetic and acoustic Casimir forces on the pull-in voltage of electromechanical switches. It is shown that for plate separations within the micron range the acoustic term arising from pressure fluctuations can influence significantly the pull-in potential in a manner that depends on the particular roughness characteristics. The roughness at short wavelengths ($<\xi$), which is characterized by the roughness exponent H, was shown to play significant role besides the effect of long wavelength parameters w and ξ. Furthermore, the temperature influence from the acoustic term appears to play also a significant role since it leads to higher acoustic forces by its increment and therefore to lower pull-in voltages.

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